CIP/MCSP (2007/08)

Selection of exercises from Pinedo (2000), *Planning and Scheduling in Manufacturing and Services.*

Part I: chapters 2 to 7

Chapter 2

2.1. A contractor has decided to use project scheduling techniques for the construction of a building. The jobs he has to do are listed in Table 2.1.

(a) Draw the precedence constraints graph.

(b) Compute the makespan of the project.

(c) If it would be possible to shorten one of the jobs by one week, which job should be shortened?

Job	*Description of Job*	*Duration*	*Immediate Predecessor(s)*
1	Excavation	4 weeks	–
2	Foundations	2 weeks	1
3	Floor Joists	3 weeks	2
4	Exterior Plumbing	3 weeks	1
5	Floor	2 weeks	3,4
6	Power On	1 weeks	2
7	Walls	10 weeks	5
8	Wiring	2 weeks	6,7
9	Communication Lines	1 weeks	8
10	Inside Plumbing	5 weeks	7
11	Windows	2 weeks	10
12	Doors	2 weeks	10
13	Sheetrock	3 weeks	9,10
14	interior trim	5 weeks	12,13
15	exterior trim	4 weeks	12
16	Painting	3 weeks	11,14,15
17	Carpeting	1 weeks	16
18	Inspection	1 weeks	17

Table 2.1: Contractor jobs

2.3. Consider a flexible flow shop environment. Whenever a job has to wait before it can start its processing at the next stage it is regarded as Work-In-Process (WIP), since it already has received processing on machines upstream. The costs associated with WIP are storage costs, amount of value already added, cost of capital, and so on. Explain why high WIP costs may postpone the starting times of jobs on certain machines.

2.5. Consider an environment that is prone to machine breakdowns and subject to high inflation and frequent strikes. What are the effects of these variables on,

(a) the Material Requirements Planning,

(b) the Work-In-Process levels, and

(c) the finished goods inventory levels?

Chapter 3

3.1. A consulting company has to install a brand new production planning and scheduling system for a client. This project requires the successful completion of a number of activities, as described in Table 3.1.

Activity	*Description of Activity*	*Duration*
1	Installation of new computer equipment	8 weeks
2	Testing of computer equipment	5 weeks
3	Development of the software	6 weeks
4	Recruiting of additional systems people	3 weeks
5	Manual testing of software	2 weeks
6	Training of new personnel	5 weeks
7	Orientation of new personnel	2 weeks
8	System testing	4 weeks
9	System training	7 weeks
10	Final debugging	4 weeks
11	System changeover	9 weeks

Table 3.1: Activity Description of Activity

The precedence relationships between these activities are depicted in Figure 3.2.

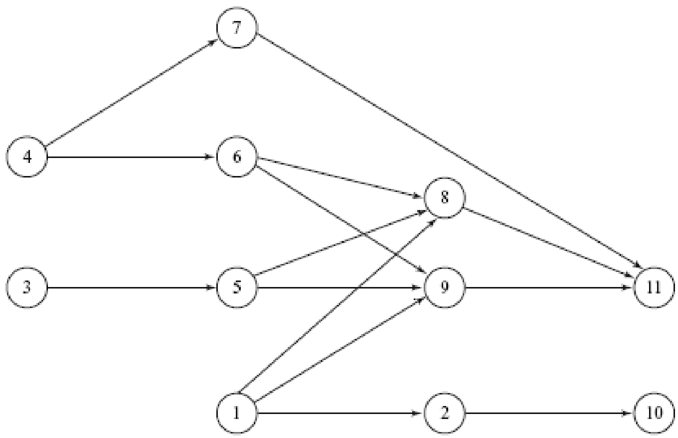

Figure 3.2: Precedence constraints of system installation

(a) Compute the makespan of the project.

(b) If it would be possible to shorten one of the activities by one week, which activity should be shortened?

Chapter 4

Example 4.1.1 (Setting up a Production Facility). Consider the problem of setting up a manufacturing facility for a new product. The project consists of eight jobs. The job descriptions and the time requirements are as follows:

Job	*Description*	*Duration* (p_j)
1	Design production tooling	4 weeks
2	Prepare manufacturing drawings	6 weeks
3	Prepare production facility for new tools and parts	10 weeks
4	Procure tooling	12 weeks
5	Procure production parts	10 weeks
6	Kit parts	2 weeks
7	Install tools	4 weeks
8	Testing	2 weeks

The precedence constraints are specified below.

Job	*Immediate Predecessors*	*Immediate Successors*
1	—	4
2	—	5
3	—	6, 7
4	1	6, 7
5	2	6
6	3, 4, 5	8
7	3, 4	8
8	6, 7	—

(a) Represent the precedence graph in the job-on-node format.

(b) Represent the precedence graph in the job-on-arc format.

(Hint: consider using a dummy job in the job-on-arc format.)

4.3. Consider the setup of the production facility described in Example 4.1.1. The durations of the 8 jobs are tabulated below.

Jobs	1	2	3	4	5	6	7	8
p_j	4	6	10	12	10	2	4	2

(a) Compute the makespan and determine the critical path(s).

(b) Suppose that the duration of job 7 can be reduced by 3 weeks to 1 week. Compute the new makespan and determine the new critical path(s).

4.4. Consider the installation of the production planning and scheduling system described in Exercise 3.1.

$Jobs$	1	2	3	4	5	6	7	8	9	10	11
p_j	8	5	6	3	2	5	2	4	7	4	9

(a) Compute the makespan and determine the critical path.

Suppose that each job can be shortened at a certain expense. The overhead cost is 6 per week (in tens of thousands of dollars). The cost functions are linear. The minimum and maximum processing times as well as the marginal costs are tabulated below.

$Jobs$	1	2	3	4	5	6	7	8	9	10	11
$p_j^{\max}$	8	5	6	3	2	5	2	4	7	4	9
$p_j^{\min}$	5	3	4	2	2	3	2	3	5	3	7
c_j^a	30	25	20	15	30	40	35	25	30	20	30
c_j	7	2	2	1	2	3	4	4	4	5	4

(b) Apply Algorithm 4.4.1 to this instance.

(c) Verify whether the solution obtained in (b) is optimal.

4.6. Consider again Example 4.1.1. Suppose it is possible to add resources to the various jobs on the critical paths in order to reduce the makespan. The overhead cost *co* per unit time is 6. The costs are linear and the marginal costs are tabulated below.

$Jobs$	1	2	3	4	5	6	7	8
$p_j^{\max}$	4	6	10	12	10	2	4	2
$p_j^{\min}$	2	5	7	10	8	1	2	1
c_j^a	20	25	20	15	30	40	35	25
c_j	4	3	4	2	3	2	4	4

(a) Determine the processing times and the makespan of the solution with the total minimum cost. Is the solution unique?
(b) Plot the relationship between the total cost of the project and the makespan.

4.8. Consider the following PERT version of the installation of the production planning and scheduling system described in Exercises 3.1 and 4.4.

$Jobs$	1	2	3	4	5	6	7	8	9	10	11
p_j^a	2	1	5	2	1	4	1	1	6	1	8
p_j^m	8	5	6	3	2	5	2	4	7	4	9
p_j^b	14	9	7	4	3	6	3	7	8	7	10

(a) Rank the paths according to the means of their total processing time.

(b) Rank the paths according to the variance in their total processing times. Which path has the highest variance? Which path has the lowest variance?

(c) Compute the probability that the makespan is larger than 27 following the standard PERT procedure.

(d) Compute the probability that the makespan is larger than 27 by considering only the path with the largest variance.

Chapter 5

5.3. Consider 6 machines in parallel and 13 jobs. The processing times of the 13 jobs are tabulated below.

$jobs$	1	2	3	4	5	6	7	8	9	10	11	12	13
p_j	6	6	6	7	7	8	8	9	9	10	10	11	11

(a) Compute the makespan under LPT.

(b) Find the optimal schedule.

5.6. Consider the following heuristic for the job shop problem with no recirculation and the makespan objective. Each time a machine is freed, select the job (among those immediately available for processing on the machine) with the longest *total* remaining processing (including its processing on the machine freed). If at any point in time more than one machine is freed, consider first the machine with the largest remaining workload.

Apply this heuristic to the following example with four machines and three jobs. The route, i.e., the machine sequence, as well as the processing times are presented in the table below.

jobs	*machine sequence*	*processing times*
1	$1, 2, 3$	$p_{11} = 10, \quad p_{21} = 8, \quad p_{31} = 4$
2	$2, 1, 4, 3$	$p_{22} = 8, \quad p_{12} = 3, \quad p_{42} = 5, \quad p_{32} = 6$
3	$1, 2, 4$	$p_{13} = 4, \quad p_{23} = 7, \quad p_{43} = 3$

5.7. Apply the heuristic described in Exercise 5.6 to the following instance with the makespan objective.

jobs	*machine sequence*	*processing times*
1	1,2,3,4	$p_{11} = 9, \quad p_{21} = 8, \quad p_{31} = 4, \quad p_{41} = 4$
2	1,2,4,3	$p_{12} = 5, \quad p_{22} = 6, \quad p_{42} = 3, \quad p_{32} = 6$
3	3,1,2,4	$p_{33} = 10, \quad p_{13} = 4, \quad p_{23} = 9, \quad p_{43} = 2$

5.8. Consider the instance in Exercise 5.7.

(a) Apply the Shifting Bottleneck heuristic to this instance (doing the computation by hand).

(b) Compare your result with the result of the shifting bottleneck routine in the LEKIN system.

5.9. Consider the following two machine job shop with 10 jobs. All jobs have to be processed first on machine 1 and then on machine 2. (This implies that the two machine job shop is actually a two machine flow shop).

jobs	1	2	3	4	5	6	7	8	9	10	11
p_{1j}	3	6	4	3	4	2	7	5	5	6	12
p_{2j}	4	5	5	2	3	3	6	6	4	7	2

(a) Apply the heuristic described in Exercise 5.6 to this two machine job shop.

(b) Construct now a schedule as follows. The jobs have to go through the second machine in the same sequence as they go through the first machine. A job whose processing time on machine 1 is shorter than its processing time on machine 2 must precede each job whose processing time on machine 1 is longer than its processing time on machine 2. The jobs with a shorter processing time on machine 1 are sequenced in increasing order of their processing times on machine 1. The jobs with a shorter processing time on machine 2 follow in decreasing order of their processing times on machine 2. (This rule is usually referred to as *Johnson's Rule.*)

(c) Compare the schedule obtained with Johnson's rule to the schedule obtained under (a).

5.10. Apply the shifting bottleneck heuristic to the two machine flow shop instance in Exercise 5.9. Compare the resulting schedule with the schedules obtained in Exercise 5.9.

Chapter 6

6.2. Consider the model discussed in Section 6.2 with 4 machines and an MPS of 4 jobs.

Jobs	1	2	3	4
p_{1j}	6	4	6	8
p_{2j}	2	10	4	6
p_{3j}	4	8	0	2
p_{4j}	8	2	6	6

(a) Apply the unweighted PF heuristic to find a cyclic schedule. Choose job 1 as the initial job and compute the MPS cycle time.
(b) Apply again the unweighted PF heuristic. Choose job 2 as the initial job and compute the MPS cycle time.
(c) Find the optimal schedule.

6.3. Consider the same problem as in the previous exercise.

(a) Apply a weighted PF heuristic to find a cyclic schedule. Choose the weights associated with machines 1, 2, 3, 4 as 2, 2, 1, 2, respectively. Select job 1 as the initial job.

(b) Apply again a weighted PF heuristic but now with weights 3, 3, 1, 3. Select again job 1 as the initial job.

(c) Repeat again (a) and (b) but select job 2 as the initial job.

(d) Compare the impact of the weights on the heuristic's performance with the impact of the selection of the first job on the performance.

6.5. In order to model a paced assembly line consider a single machine and n jobs. The processing times of each one of the jobs is 1. Job j has a due date d_j and a weight w_j. If job j is completed after its due date, then a penalty w_jT_j is incurred. There are sequence dependent setup costs c_{jk} but no setup times. The sequence dependent setup costs are determined as follows: job j has an attribute a_j that can be either 0 or 1. Most jobs have an attribute value a_j equal to 0. If there are two jobs with a_j equal to 1 sequenced in such a way that there are less than 3 jobs with a_j equal to 0 in between, then a penalty cost $c_1 = 3$ is incurred. (This implies that this attribute a_j corresponds to a capacity constrained operation).

(a) Design an algorithm for finding a sequence with minimum cost.

(*Hint:* Consider, for example, a composite dispatching rule followed by a local search; see Appendix C.)

(b) Apply the algorithm developed to the following instance.

Jobs	1	2	3	4	5	6	7	8	9	10
a_j	0	1	1	0	0	0	1	0	0	0
d_j	∞	2	∞	1	∞	5	6	∞	2	∞
w_j	0	4	0	1	0	3	4	0	3	0

Compute the total cost of the sequence.

6.6. Apply the GS heuristic to the instance in Exercise 6.5. Compare the result with that obtained in the previous exercise.

6.7. Consider the model in Exercise 6.5. Now job j has two attributes a_j and b_j. The a_j attribute is the same as in Exercise 6.5. The b_j values also can be either 0 or 1. If two jobs with b_j value equal to 1 are positioned in such a way that there are 4 or less jobs with b_j value equal to 0 in between, then a penalty cost $c_2 = 3$ is incurred. (This situation corresponds to a line with two capacity constrained operations).

(a) Describe how the algorithm of Exercise 6.5 has to be modified to take this generalization into account.

(b) Apply your algorithm to the instance below.

Chapter 7

7.1. Consider four different products with the following demand rates, production rates, holding costs and setup costs.

items	1	2	3	4
D_j	50	50	60	60
Q_j	400	500	500	400
h_j	20	20	30	70
c_j	2000	1000	1000	100

(a) Find the optimal rotation schedule. Determine its cycle length, and the total idle time.

(b) Suppose now that item 4 can be produced many times during a cycle. Items 1, 2 and 3 still can be produced only once during a cycle. Find the optimal production schedule. How does the optimal cycle length compare with the original optimal cycle length?

7.2. Consider two identical machines in parallel. Four items have to be produced.

items	1	2	3	4
D_j	50	50	60	60
Q_j	400	500	500	400
h_j	20	20	30	70
c_j	2000	1000	1000	100

(a) Find the optimal rotation schedule assuming that the cycle lengths of the two machines have to be the same. Compute the total average cost per unit time.

(b) Find the optimal rotation schedules of the two machines assuming the cycle lengths of the two machines do not have to be the same (determine which items have to be combined with one another on the same machine to obtain the best result). Compute the average cost per unit time and compare the result with the result found under (a).

7.3. Consider the following two stage production process in a paper mill with a downstream converting operation. At the first stage there is a single paper machine. The output of this operation consists of large rolls of paper. The second stage is a single machine cutting operation that produces cutsize paper. To simplify the problem assume that only two items have to be produced. Also, each item that comes out of the second stage corresponds to one of the items that comes out of the first stage. The production rates, setup costs and holding costs are different at the two stages. In the table below Q_{ij} denotes the production rate of item j at stage i, c_{ij} the setup cost of item j at stage i, and h_{ij} the holding cost of item j after processing at stage i (so h_{1j} denotes the holding cost of keeping item j in inventory in between the two stages, while h_{2j} denotes the holding cost of item j as a finished good).

items	1	2
D_j	100	50
Q_{1j}	400	400
Q_{2j}	600	1000
h_{1j}	20	20
h_{2j}	60	80
c_{1j}	3000	2500
c_{2j}	1000	1250

The schedules at both stages have to be rotation schedules (i.e., it is, for example, not allowed to produce item 1 at the first stage for a while, leave the machine idle for some time, produce item 1 again, and then item 2).

(a) Assuming that the cycle length x of the two stages have to be the same, what is the cycle length with the minimum total cost?

(b) Assume that the cycle lengths at the two stages are allowed to be different. Determine the optimal cycle lengths of the two stages.

7.7. Consider a paper mill with two paper machines. There are 5 different types of paper that have to be produced. Items 1 and 2 have to be produced on machine 1 and item 3 has to be produced on machine 2. Items 4 and 5 can be produced on either one of the two machines.

items	1	2	3	4	5
D_j	60	60	80	80	100
Q_j	200	200	300	300	400
h_j	20	30	40	20	20
c_j	3000	2000	800	4000	1500

(a) Determine the optimal rotation schedule assuming that the cycle lengths have to be the same.

(b) Determine the optimal rotation schedules assuming the cycle lengths do not have to be the same.

(c) Determine the optimal schedule if the schedule on machine 1 has to be a rotation schedule and the schedule on machine 2 may be an arbitrary schedule.

7.8. Consider the following generalization of the paper making facility of Exercise 7.3. Again, there are two stages. The entire facility produces three different items. However, the paper machine at the first stage produces only two different intermediate products. One of the intermediate products that comes out of stage 1 is used at stage 2 to produce items 1 and 2. (This implies that the data corresponding to items 1 and 2 regarding stage 1 are the same.) The other intermediate product that comes out of stage 1 is converted at stage 2 into item 3.

items	1	2	3
D_j	50	70	100
Q_{1j}	250	250	300
Q_{2j}	200	400	300
h_{1j}	30	30	40
h_{2j}	40	20	30
c_{1j}	2000	2000	900
c_{2j}	1000	3000	2000

Determine the optimal rotation schedule and its cycle length.